1. Imagination/Inspiration

At least in some circles, math and art are seen as disciplines which share a common claim to a certain level of abstract thought - more or less divorced (at least theoretically) from empirical evidence they are expressions of the 'pure' human mind. As such, notions of imagination and inspiration are often associated with the synthesis of works of the respective disciplines. Although it may be true that both math and art enjoy some distance from the 'mundane' considerations of day-to-day life (relative to other disciplines), I would say that much of what constitutes a successful mathematician/artist is maybe more the ability to channel this energy into consistent production. I'm often amazed (intimidated?) by the number of papers, conferences, etc. that the most successful mathematicians are able to contribute to - these are people who are able to takes their thoughts and convert them into a paper/presence in a very efficient manner. This is not to say that the quality of the idea is distilled in any way (this is clearly not the case), but more that these people are able to overcome a somewhat natural aversion to the 'compromise' of making the thoughts concrete. We often feel inclined to differentiate between the 'pure' thoughts of a mathematician/artist and the actual instantiation/production of them (papers, painting, etc.) but I feel like it's a blurry boundary and really these are often one in the same.

2. Process/Rules

Perhaps a common conception is that mathematicians have a certain list of 'obvious' (and static) axioms/definitions that we then bump into each other to produce new results. Although this is to some extent an accurate depiction of the process of math research, things are of course more sophisticated. As mathematicians, we've all heard some form of the old adage that we 'know what the theorems should be, we just need the right definitions'. The sentiment is somewhat tongue-in-cheek, but I do think it describes the process of math research more accurately than those outside of math might otherwise imagine. Most of the 'good' definitions in math have evolved (and continue to evolve) over many years and have involved the best minds in history - notions of manifolds, matroids, and schemes come to mind, and even more 'down to earth' concepts such as groups and topological spaces are good examples. This is perhaps analogous to the (misconceptions of the) processes involved in art practice: it is not so much that artists have a set of tools and techniques from which to produce their works of art, but rather they seek to define these techniques in a way that they and others can then incorporate into their works.

Of course once the definitions are established one does need to deduce some things from them (the conjectures that should be true actually do need to be proved) and this process does account for a good chunk of the research process. Zack mentioned the use of metaphor is his research process - my impression is that he's referring to the use of language outside mathematics to describe some more familiar aspects of e.g. an involved alegbro-geometric concept. This is of course a powerful way to understand and gain intuition into a certain problem. But I think an equally relevant use of metaphor in math research comes in the form of metaphor *within* mathematics - that is, using concepts and constructions from one discipline (within math) and applying them to other areas where they apparently have no place being. The surprising utility (in some cases) of these metaphors account for some of the most important and 'deepest' mathematical research.

One of the most interesting (for me) themes coming out of our post presentation discussions was the notion of the local vs. global aspect of process in both math and art creation. I believe it was Eric, who, in his description of his artistic process, mentioned something to the effect that each step in making his paintings was governed by some rule, a rule that he either invented for himself or else by intuition presented itself to him. I think this 'following your nose' process according to local rules is a good metaphor for certain kinds of mathematical research. When one goes about proving a statement, we often have a certain 'checklist' of tools/arguments that one applies at a certain step - although the global statement (e.g. the conclusion of the theorem) is difficult to see, the local steps are there. In both math and art we have (through experience) grown accustomed to these local steps/procedures that govern our process on the small scale; these rules guide our hand and provide us with intuition. This intuition is continually enhanced (the checklist gets longer) as we make small deviations from previous efforts. In Lun-Yi's discussion of his paintings of color fields, he mentioned the consuming/frustrating process of continually changing patches of colors to the point where he began to anticipate the effect of making a particular small change. These were perhaps the local rules that he was learning that began to guide his hand in subsequent efforts. Although he might not be paintings color fields these days, I would say that these lessons still do inform his mark making.

Although the local rules are somewhat automatic and part of the language, the most effective global efforts in math/art involve steps that are truly novel and outside the expected checklist. The proof of a theorem often involves a 'key lemma', something that one might not try if he/she were presented with the task. The complete proof might involve a (perhaps complicated) sequence of expected local moves/rules that revolve around this insight. The proofs that are considered most interesting are often the ones that involve at least some sort of departure from the expected sequence of local steps.

3. Notation

As Zack mentioned, there is a strong emphasis in mathematics on good notation. Some of the best talks and papers that we come across are those that use the language in a efficient and streamlined manner. Zack pointed out the learned skill of literally writing down funny symbols like ξ, but also mentioned the need for notating/representing a concept in a meaningful way. The quest for good notation and the 'right' definitions has in some sense led to the creation of entire branches of mathematics. Perhaps the archetypal example is the branch of mathematics known as category theory (the granddaddy of the short exact sequence?), which some would say has no theorems at all, i.e. just a bunch of definitions and notation. But category theory has developed hand in hand with some of the most powerful mathematics of the contemporary era, and today it is a language used by mathematicians in diverse fields as the 'right way' to talk about various constructions, etc. Although it's still common to disregard the work of a mathematician by saying it's 'just notation' or 'all definitions', I would say it's often a term of endearment in a strange way.

Perhaps this is analogous to the experience of an artist who has to negotiate the space between that of a 'technician' and that of a true 'artist'. I would guess that very few of the historically successful artists lack what one would consider technical skill, although again a common way to dismiss a work of art is to describe it as technically strong but 'lacking in concept'; here one encounters the often controversial distinction between the 'artist' and the 'craftsperson'. This was especially the case in the mid to late 20th century as the emphasis in art moved away from that of a mastery of technique to that of conceptual art (here we see works of art being physically produced not by the artist him/herself but by a team of craftspeople hired to carry out the vision of the 'artist'). I would say that even this distinction seemed to fade away in the late 20th century with the advent of post-modern (and post-PM) movements, where we see a certain reclamation of the aesthetics of pure technical skill (is that true?)

4. Visualization/Representation

One thing that math and art (and other fields) have in common is that in both cases we are using a sort of language to represent abstract concepts In mathematics it is typical that one works with an object/concept that has a rather complicated definition - it is the long list of conditions that one uses to formally prove something about it. However, often the most effective way to communicate the 'meaning' of the concept is to provide a one or two illustrative examples (often the ones that motivated the definition). If we all agree that these examples are somehow captured/unified by the axioms involved, then it is a object that gains some value.

The most common question that came up in the post presentation dialogues (and elsewhere) was some form of 'how do mathematicians 'visualize' higher dimensions?' For the mathematicians I don't think this question really makes sense because in the cases where one deals with higher dimensions the concept is well-defined (usually according to some formalized notion of 'degree of freedom'). Depending on your definition, you formally work in higher dimensions just as you would in 2 or 3; of course geometric intuition comes into play in certain circumstances, but no one is really 'seeing' $\mathbb{R}^{12}$. Perhaps the better question is 'how do mathematicians define dimension'?

In my effort to address the original question (I was anticipating it), I tried to present the concept of a polytope in a way that might give some insight into these 'higher dimensional' things. One the one hand a polytope is a space that is (by definition) the intersection of a bunch of half spaces in high dimensional space (OK, hard to see), but also contains the extra information regarding the way points, edges, faces, etc. are related to each other. This built-in (and often recursive) structure of the faces gives you a way to see what's going on - the example I used was the 4 dimensional cube that can be 'unfolded' in the same way one unfolds the (usual) 3-cube into a flat collection of squares glued in a certain way. (Polytopes can also be equivalently described as the convex hull of finitely many points but I would argue this is even 'harder' to see).

5. Parameters

As Lun-Yi mentioned, the notion of a parameter can mean many things depending on context (vernacular vs technical), but even within mathematics I would say its meaning is somewhat blurred. It's true that folks often use the word 'parameter' interchangeably with 'variable', i.e. the things you stick into a formula/function (some would argue that the parameters stay fixed while the variables are the ones that change, so that the parameters are actually part of the function's definition). But I think the more interesting notion of a 'parameterization' involves the idea of taking a collection of things (points of a curve, curves themselves with a particular property, triangulations of a polygon, etc.) and ascribing/mapping to them in a bijective way the points of some *other* object/space. The idea is then that we can work with this meta-object to reveal something interesting about the original collection that we had in mind.

Here is a simple example: let's take *the points* of a circle in the x-y plane with radius 1. Maybe the first way we learn to describe this algebraically is with the equation $x^2 + y^2 = 1$. OK, but now I want to think about this circle as a certain piece of (say) string that has been placed on the plane tracing the circular shape, ie. I want to think about the points on the circle and coming from points of a line segment (string). We do this by 'parameterizing' the circular according to $(\cos t, \sin t)$ with $0 \leq t < 2\pi$. What we mean by this is that we take a piece of (straight) string of length 2π and place it on the plane according to: the piece at position t should go at the point $(\cos t, \sin t)$ in the plane. Although a simple example, I would say that this notion is very important/powerful in many branches of math. The classification of curves into rational vs. elliptic and higher genus runs along these lines.

More sophisticated examples of parameterization involve the notion of taking a collection of objects in a certain category and ascribing *to that collection* the properties that the objects themselves enjoy - the ultimate example perhaps being the moduli spaces that Sándor addressed. In my presentation I tried to describe how one can similarly take a collection of 'bracketings' of a string of letters and think of them as the points of a polytope. To make this more of a moduli space type picture, one can generalize this and give the collection of (certain) maps between polytopes P and Q the structure of a polytope. In both cases, the idea is that we are given a discrete collection of objects that have certain relationships between, and we are able to ascribe to them the structure of a geometric object (a polytope) in a way that the face structure of the polytope encodes that given relationship.